

# Chapter 4

## Congruent Triangles

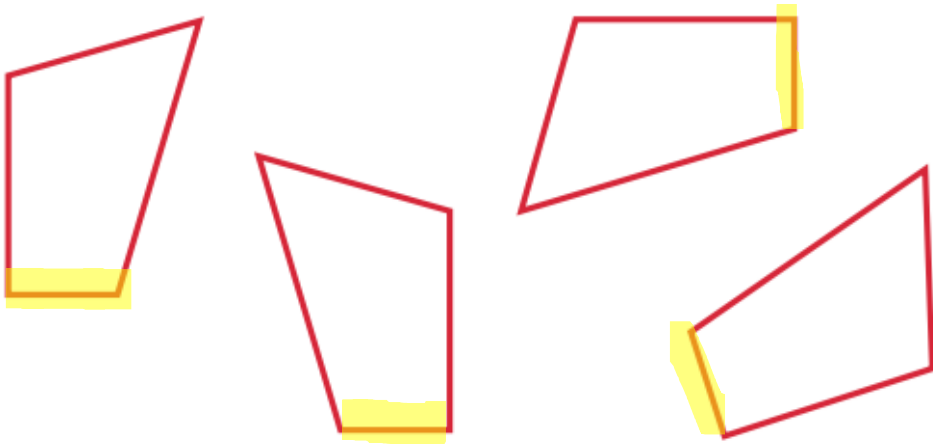
# Section 2

## Congruence and Triangles

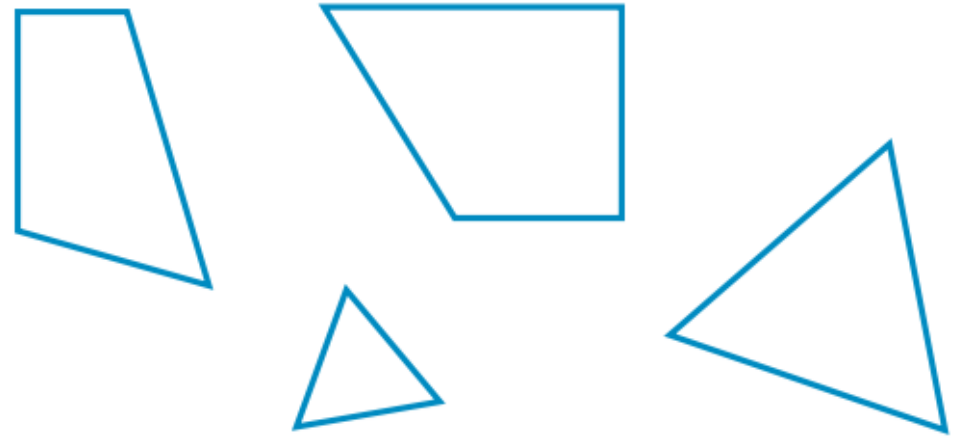
## GOAL 1: Identifying Congruent Figures

Two geometric figures are congruent if they have the exact same size and shape. Each of the red figures is congruent to the other red figures. None of the blue figures are congruent to the other blue figures.

**Congruent**



**Not congruent**



When two figures are congruent, there is a correspondence between their angles and sides such that corresponding angles are congruent and corresponding sides are congruent. For the triangles below, you can write  $\triangle ABC \cong \triangle PQR$  which is read “triangle ABC is congruent to triangle PQR.” The notation shows the congruence and the correspondence.

**Corresponding angles**

$$\angle A \cong \angle P$$

$$\angle B \cong \angle Q$$

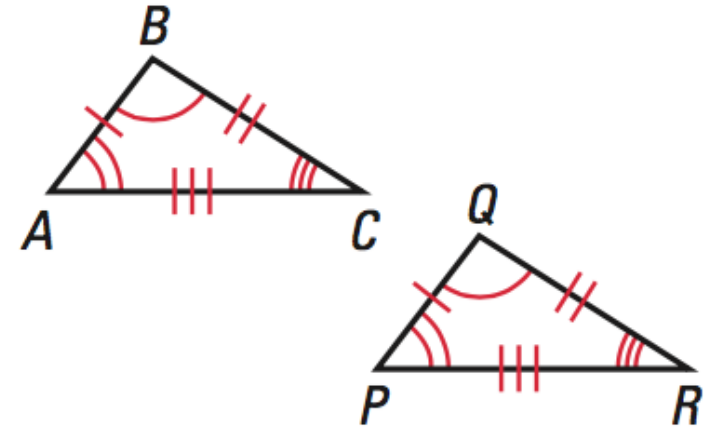
$$\angle C \cong \angle R$$

**Corresponding sides**

$$\overline{AB} \cong \overline{PQ}$$

$$\overline{BC} \cong \overline{QR}$$

$$\overline{CA} \cong \overline{RP}$$



There is more than one way to write a congruence statement, but it is important to **list the corresponding angles in the same order**. For example, you can also write  $\triangle BCA \cong \triangle QRP$ .

## Example 1: Naming Congruent Parts

The congruent triangles represent the triangles in the photo. Write a **congruence statement**. Identify all parts of congruent corresponding parts.

**Tri. FED cong. Tri. TSR**

Angles:

$\angle F \text{ cong. } \angle T$

$\angle E \text{ cong. } \angle S$

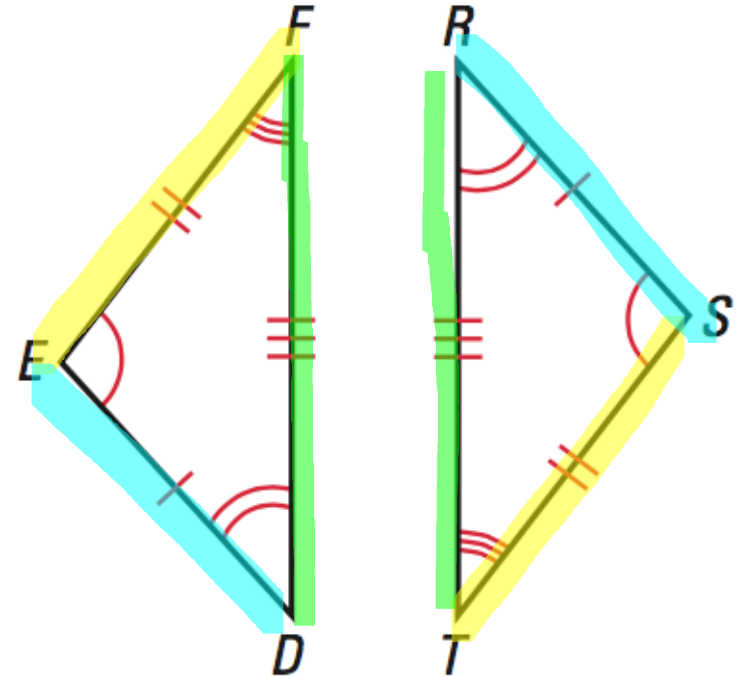
$\angle D \text{ cong. } \angle R$

Sides:

**FE cong. TS**

**ED cong. SR**

**DF cong. RT**



## Example 2: Using Properties of Congruent Figures

In the diagram,  $NPLM \cong EFGH$ .

a) Find the value of  $x$ .

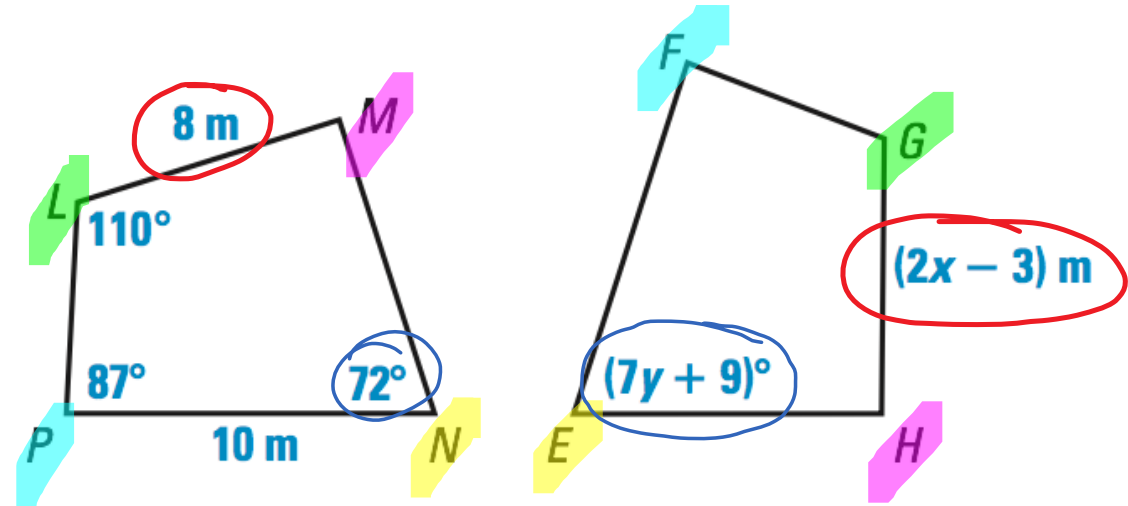
$$\begin{array}{r} 2x - 3 = 8 \\ +3 \quad +3 \end{array}$$

$$\frac{2x}{2} = \frac{11}{2} \rightarrow x = 5.5$$

b) Find the value of  $y$ .

$$\begin{array}{r} 7y + 9 = 72 \\ -9 \quad -9 \end{array}$$

$$\frac{7y}{7} = \frac{63}{7} \rightarrow y = 9$$



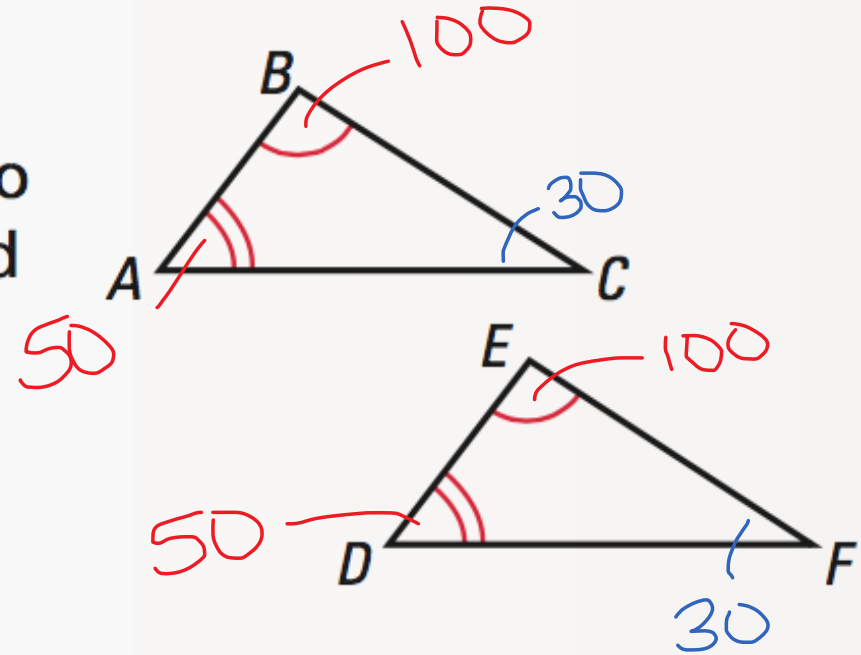
The Third Angles Theorem below follows from the Triangle Sum Theorem. You are asked to prove the Third Angles Theorem in Exercise 35.

## THEOREM

### THEOREM 4.3 *Third Angles Theorem*

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

If  $\angle A \cong \angle D$  and  $\angle B \cong \angle E$ ,  
then  $\angle C \cong \angle F$ .



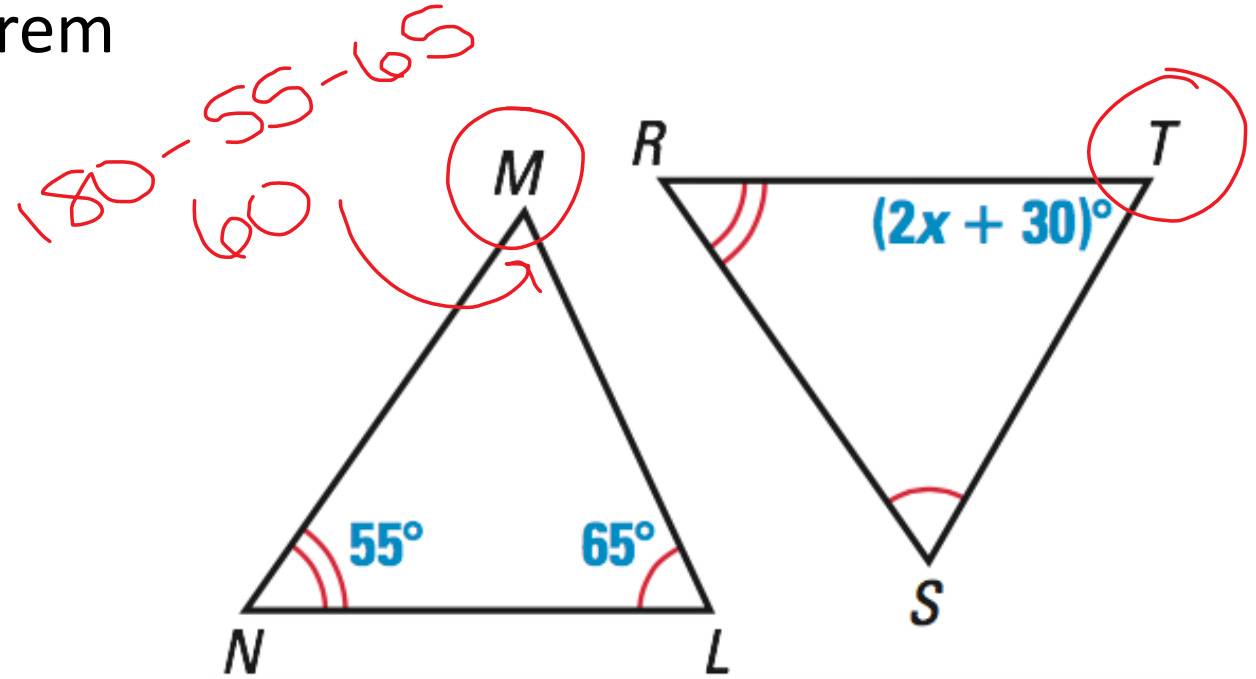
### Example 3: Using the Third Angles Theorem

Find the value of  $x$ .

$$\begin{array}{r} 2x + 30 = 60 \\ -30 \quad -30 \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{30}{2}$$

$$x = 15$$





## GOAL 2: Proving Triangles are Congruent

### Example 4: Determining Whether Triangles are Congruent

Decide whether the triangles are congruent. Justify your reasoning.

Sides:

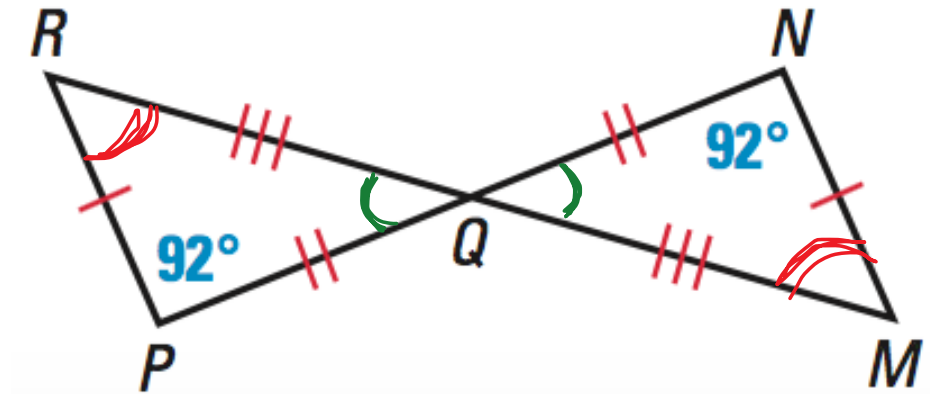
QM cong. QR, QN cong. QP, PR cong. NM

Angles:

$\angle P$  cong.  $\angle N$

$\angle PQR$  cong.  $\angle NQM$  (vertical  $\angle$ s)

$\angle R$  cong.  $\angle M$  (3<sup>rd</sup>  $\angle$ s theorem)

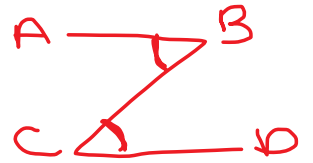
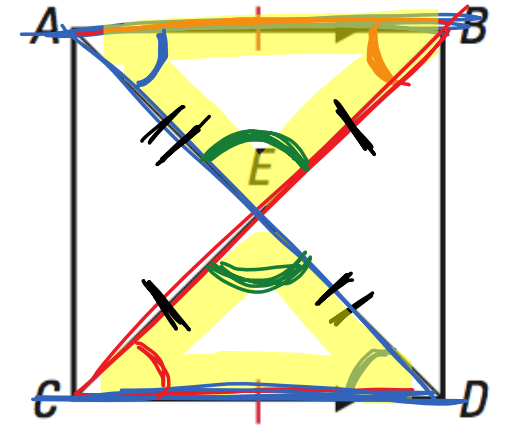
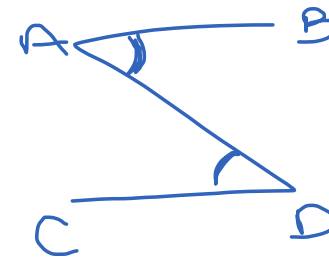


→ Yes they are congruent

## Example 5: Proving Two Triangles are Congruent

Use the diagram to the right. Prove that  $\triangle AEB \cong \triangle DEC$ .

(Given:  $AB \cong CD$ ,  $AB \parallel CD$ , E is the midpoint of BC and AD)



Statements

- 1)
- 2)  $\angle BAD \cong \angle CDA$ ;  $\angle ABC \cong \angle DCB$
- 3)  $\angle AEB \cong \angle DEC$
- 4)  $BE \cong CE$ ;  $AE \cong DE$
- 5)  $\triangle AEB \cong \triangle DEC$

Reasons

- 1) Given
- 2) Alt. Int.  $\angle$ s
- 3) 3<sup>rd</sup>  $\angle$ s Theorem (vertical  $\angle$ s)
- 4) Def. of midpoint
- 5) Def. of cong. triangles

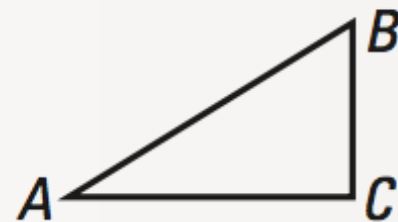
In this lesson, you have learned to prove that two triangles are congruent by the *definition of congruence*—that is, by showing that all pairs of corresponding angles and corresponding sides are congruent. In upcoming lessons, you will learn more efficient ways of proving that triangles are congruent. The properties below will be useful in such proofs.

## THEOREM

### THEOREM 4.4 *Properties of Congruent Triangles*

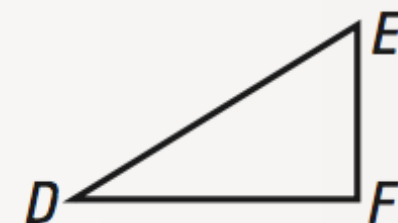
#### REFLEXIVE PROPERTY OF CONGRUENT TRIANGLES

Every triangle is congruent to itself.



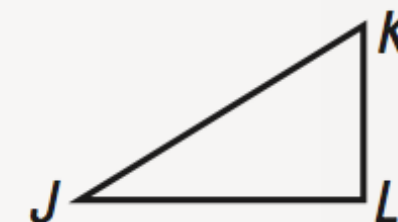
#### SYMMETRIC PROPERTY OF CONGRUENT TRIANGLES

If  $\triangle ABC \cong \triangle DEF$ , then  $\triangle DEF \cong \triangle ABC$ .



#### TRANSITIVE PROPERTY OF CONGRUENT TRIANGLES

If  $\triangle ABC \cong \triangle DEF$  and  $\triangle DEF \cong \triangle JKL$ , then  $\triangle ABC \cong \triangle JKL$ .



EXIT SLIP